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An Advanced Controller for Robotic Manipulator

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Abstract

This paper proposes higher order sliding mode controller for robotic manipulator. The scheme is used to compensate for the influence of unmodeled dynamics and to reduce chattering. Simulation results show that the proposed controller gives better performance compared to fuzzy sliding mode control in the face of uncertain system parameters and external disturbances.

Keywords: Fuzzy sliding mode, higher order sliding mode, robotic manipulator.

Introduction

Classical sliding mode control (CSMC) is a powerful scheme for nonlinear systems with uncertainty [1]. However this control scheme suffers from some problems like chattering. In order to guarantee the stability of the sliding mode system, the boundary of the uncertainty has to be estimated. Chattering will occur due to finite switching of switched controller in real time systems. Several approaches for reducing the chattering have been proposed, among which the well known one is to apply a saturation function [1] to the control gain when the sliding surface is within a boundary of the sliding hyper-plane. This approach leads to tracking to within a guaranteed precision (rather than perfect tracking).

An alternative solution to reduce chattering phenomena is applying fuzzy logic sliding mode control approach or higher order sliding mode control approach [2]. Both approaches can reduce the chattering effect but the error performance is improved in higher order sliding mode approach. Fuzzy control offers ways to implement simple and robust solutions that can cope with a wide range of system parameters with disturbances [3]. Fuzzy logic has proven to be a potent tool in the sliding mode control of time-invariant linear systems as well as time-varying nonlinear systems [4, 5]. This combination of conventional theory and fuzzy control has received a great deal of attention [6]. Fuzzy control has been demonstrated to provide a powerful tool for fine tuning of control algorithms based on conventional control theoretic approaches [7].

In this paper, a higher order sliding mode controller (HOSMC) [9, 10] for a robotic manipulator is proposed. The higher order switching gains are incorporated in the sliding mode controller to accelerate the state trajectories toward the sliding hyper plane. To demonstrate its effectiveness, the proposed HOSMC algorithm is applied to simulate two-link robot manipulator.

This paper is organized as follows. Section II gives some background on robot dynamics and Classical Sliding Mode Control (CSMC). Section III gives sliding mode control with fuzzy logic implementation. Section IV presents higher order sliding mode control algorithm. In section V, the simulation studies for two-degree of freedom robot is illustrated and conclusions are presented in Section VI.

Classical Sliding Mode Control for Robotic Manipulator

This section briefly reviews the basic concepts sliding mode control for robotic manipulators.

A. *Robot Manipulator Dynamics:*

Using a Lagrangian formulation, the generalized forces for a n-link manipulator can be expressed as a second-order nonlinear differential equation [3, 4].

$$\mathbf{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau} \quad (1)$$

where, $\mathbf{q}$, $\mathbf{\dot{q}}$ and $\mathbf{\ddot{q}} \in \mathbb{R}^n$ are the joint position, velocity and acceleration vector respectively. $\mathbf{M(q)}$ is an *n x n* inertial matrix, $\mathbf{C(q,\dot{q})}$ is an *n x n* matrix of Coriolis and centrifugal forces and $\mathbf{G(q)}$ is an *n x 1* gravity vector.

Properties for dynamics of robot manipulators:

Property 1: The matrix $\mathbf{M(q)}$ is symmetric, positive definite and bounded above and below its inverse exists and is positive definite and bounded.

Property 2: The matrix $\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is skew, which suggests $\dot{\mathbf{M}} = \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})^T$ and $x^T\left[\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\right]x = 0$, $\forall x \in \mathbb{R}^n$.

Property 3: The dynamic structure (1) is linear in terms of suitable selected set of robot and load parameters, i.e. $\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{w}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\boldsymbol{\varphi}$ where $\mathbf{w}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\boldsymbol{\varphi} \in \mathbf{R}^{\mathbf{nxr}}$ is a nonlinear function of the manipulator structural dynamics and $\boldsymbol{\varphi} \in \mathbf{R}^{\mathbf{r}}$ **is** the vector containing the unknown manipulator parameters which includes the numerical values of the masses and moment of inertia of the links and the payloads and the link dimension.

B. Classical Sliding Mode Control for Robotic Manipulator:

The control objective is to drive the joint position $\mathbf{q}$ to the desired position $\mathbf{q_d}$. Define the tracking error

$$\mathbf{e} = \mathbf{q} - \mathbf{q_d} \quad (2)$$

Define the sliding surface

$$\mathbf{s} = \dot{\mathbf{e}} + \boldsymbol{\lambda}\mathbf{e} \quad (3)$$

where $\boldsymbol{\lambda} = diag\left[\lambda_1.....\lambda_i.....\lambda_n\right]$ in which λ_i is a positive constant.

The control objective can be achieved by choosing the control input so that the sliding surface satisfies the sufficient condition [1].

$$\frac{1}{2}\frac{d}{dt}s_i^2 \le -\eta_i\left|s_i\right| \quad (4)$$

where "η_i" is a positive constant Equation (4) indicates that the energy of should decay as long as is not zero. To setup τ, define the reference state

$$\dot{\mathbf{q}}_\mathbf{r} = \dot{\mathbf{q}} - \mathbf{s} = \dot{\mathbf{q}}_\mathbf{d} - \boldsymbol{\lambda}\mathbf{e}$$
$$\ddot{\mathbf{q}}_\mathbf{r} = \ddot{\mathbf{q}} - \dot{\mathbf{s}} = \ddot{\mathbf{q}}_\mathbf{d} - \boldsymbol{\lambda}\dot{\mathbf{e}} \quad (5)$$

Choose the control input $\boldsymbol{\tau}$

$$\boldsymbol{\tau} = \hat{\boldsymbol{\tau}} - \mathbf{K}\,\mathrm{sgn}(\mathbf{s})$$

$$\hat{\boldsymbol{\tau}} = \hat{\mathbf{M}}\ddot{\mathbf{q}}_\mathbf{r} + \hat{\mathbf{C}}\dot{\mathbf{q}}_\mathbf{r} + \hat{\mathbf{G}} - \mathbf{A}\mathbf{s} \quad (6)$$

where $\hat{\mathbf{M}}$, $\hat{\mathbf{C}}$ and $\hat{\mathbf{G}}$ are the estimation matrices of **M,** ***C*** and ***G*** respectively.

$\mathbf{K} = diag\left[K_{11},...K_{ii},...K_{nn}\right]$ is a diagonal positive definite matrix in which K_{ii} is a positive constant and $\mathbf{A} = diag\left[a_1,...a_i,...a_n\right]$ is also a diagonal positive definite matrix in which a_i is a positive constant.

Fuzzy Sliding Mode Control

The simulation results for classical sliding mode control are shown in section V, where chattering effect is observed. The chattering is caused by the constant value of $\mathbf{K}$ and the discontinuous function $\mathrm{sgn}(\mathbf{s})$ .one way to eliminate chattering is to replace $\mathrm{sgn}(\mathbf{s})$ by $sat(\mathbf{s}/\phi)$ but this approach may introduce steady-state error into the sliding surface [1]. In this section, a fuzzy control gain $\mathbf{k}$ is applied to construct fuzzy sliding mode controller [4].

A. Introduction of Fuzzy Systems:

Fig. 1 shows the block diagram of a typical fuzzy system. Usually, a fuzzy system has one or more inputs and a single output. A multiple-output system can be considered as a combination of several single-output systems [2]. There are four basic parts in a fuzzy system. The fuzzification and defuzzification are the interface between the fuzzy systems and the crisp systems. The rule base includes a set of rules extracted from the human experience. Each rule describes a relation between the input space and the output space. For each rule, the inference engine maps the input fuzzy sets to an output fuzzy set according to the relation defined by the rule. It then combines the fuzzy sets from all the rules in the rule base into the output fuzzy set. This output fuzzy set is translated to a crisp value output Y the defuzzification.

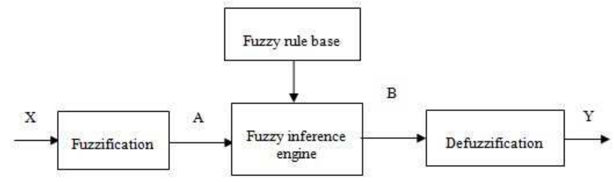

Fig.1. Diagram for a fuzzy system

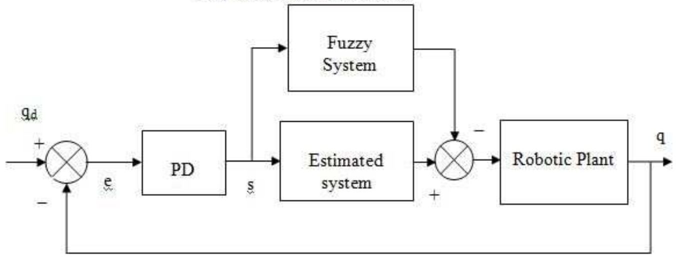

Fig.2. Fuzzy sliding mode control for robotic manipulator

All the four parts can be mathematically formulated. In this paper, by choosing singleton fuzzification, center average defuzzification, Mamdani implication in the rule base and product inference engine, the output of the fuzzy system can be written as

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$$y = \frac{\sum_{m=1}^{M} \theta^m \prod_{i=1}^{n} \mu_{A_i^m}\left(x_i^*\right)}{\sum_{m=1}^{M} \prod_{i=1}^{n} \mu_{A_i^m}\left(x_i^*\right)} = \boldsymbol{\theta}^T \boldsymbol{\psi}(x) \qquad (7)$$

where $\boldsymbol{\theta} = \left[\theta^1, \ldots \theta^m, \ldots \theta^M\right]^T$ is the vector of the centers of the membership is functions of y, $\boldsymbol{\psi}(x) = \left[\psi^1(x), \ldots \psi^m(x), \ldots \psi^M(x)\right]^T$ is the vector of the height of the membership functions of y in which $\psi^m(x) = \prod_{i=1}^{n} \mu_{A_i^m}\left(x_i^*\right) \Big/ \sum_{m=1}^{M} \prod_{i=1}^{n} \mu_{A_i^m}\left(x_i^*\right)$ and M is the number of rules.

B. Fuzzy Sliding Mode Control:

Rewriting the dynamic equation of the robotic manipulator

$$\mathbf{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \boldsymbol{\tau}} \quad (8)$$

Since the chattering caused by constant gain K and discontinuous function $\operatorname{sgn}(s)$, let the control gain $\mathbf{K}\operatorname{sgn}(\mathbf{s})$ can be replaced by fuzzy controller gain $\mathbf{k}$ in the control input. The new control input is then written as

$$\boldsymbol{\tau} = \hat{\mathbf{M}}\ddot{\mathbf{q}}_{\mathbf{r}} + \hat{\mathbf{C}}\dot{\mathbf{q}}_{\mathbf{r}} + \hat{\mathbf{G}} - \mathbf{As} - \mathbf{k} \qquad (9)$$

where $\mathbf{k} = \left[k_1, \ldots k_i, \ldots k_n\right]^T$ with each k_i is estimated by individual fuzzy system.
From the knowledge of the fuzzy systems, k_i can be written as

$$k_i = \frac{\sum_{m=1}^{M} \theta_{k_i}^m \prod_{i=1}^{n} \mu_{A^m}(s_i)}{\sum_{m=1}^{M} \mu_{A^m}(s_i)} = \boldsymbol{\theta}_{k_i}^T \boldsymbol{\psi}_{k_i}(s_i) \quad (10)$$

where $\boldsymbol{\theta}_{k_i} = \left[\theta_{k_i}^1, \ldots \theta_{k_i}^m, \ldots \theta_{k_i}^M\right]^T$,

$\boldsymbol{\psi}_{k_i}(s_i) = \left[\psi_{k_i}^1(s_i), \ldots \psi_{k_i}^m(s_i), \ldots \psi_{k_i}^M(s_i)\right]^T$ and

$\psi_{k_i}^m(s_i) = \mu_{A^m}(s_i) \Big/ \sum_{m=1}^{M} \mu_{A^m}(s_i)$. θ_{k_i} is chosen as the parameter to be updated and therefore is called the parameter vector. $\psi_{k_i}(s_i)$ is called the function basis vector and can be regarded as the weight of the parameter vector.

Higher Order Sliding Mode Control

A sliding-mode controller of a new type is proposed in this note, being a feedback function of $\sigma, \dot{\sigma}, \ddot{\sigma}, \ldots, \sigma^{(r-1)}$ continuous everywhere except the manifold defined by the equations [9].

$$\sigma = \dot{\sigma} = \ddot{\sigma} = \ldots.. = \sigma^{(r-1)} = 0 \qquad (11)$$

The mode $\sigma \equiv 0$ is established after a finite-time transient. In the presence of errors in evaluation of the output σ and its derivatives, a motion in some vicinity of (11) takes place. Therefore, control is practically a continuous function of time, for the trajectory never hits the manifold (11) with r > 1.

Following are controllers with $r \le 3$ [9].

1) $u = -\alpha \operatorname{sign}\sigma$;

2) $u = -\alpha\left(\dot{\sigma} + |\sigma|^{1/2} \operatorname{sign}\sigma\right) \Big/ \left(|\dot{\sigma}| + |\sigma|^{1/2}\right)$;

3)
$$u = -\alpha\left[\ddot{\sigma} + 2\left(|\dot{\sigma}| + |\sigma|^{2/3}\right)^{-1/2}\left(\dot{\sigma} + |\sigma|^{2/3} \operatorname{sign}\sigma\right)\right] \Big/ \left[|\ddot{\sigma}| + 2\left(|\dot{\sigma}| + |\sigma|^{2/3}\right)^{1/2}\right]$$

The control is a continuous function of time everywhere except the r-sliding set (1).

A. Controller design for Robotic Manipulator:

Rewriting the dynamic equation of the robotic manipulator

$$\mathbf{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \boldsymbol{\tau}} \quad (12)$$

Define $\boldsymbol{\sigma} = \mathbf{e} = \mathbf{q} - \mathbf{q_d}$ (13)

Choose the control input $\boldsymbol{\tau}$ as

$$\boldsymbol{\tau} = \hat{\boldsymbol{\tau}} + \boldsymbol{\tau}_{\mathbf{s}} \qquad (14)$$

Where $\hat{\boldsymbol{\tau}}$ is same as chosen in Conventional Sliding Mode Control and $\boldsymbol{\tau}_{\mathbf{s}}$ is selected using r – sliding controller. Since the relative degree for robotic manipulator is 2, we chosen $\boldsymbol{\tau}_{\mathbf{s}}$ as given below.

$$\boldsymbol{\tau}_{\mathbf{s}} = -\alpha\left(\dot{\boldsymbol{\sigma}} + |\boldsymbol{\sigma}|^{1/2} \operatorname{sign}\boldsymbol{\sigma}\right) \Big/ \left(|\dot{\boldsymbol{\sigma}}| + |\boldsymbol{\sigma}|^{1/2}\right) \text{ (15).}$$

Simulation Results

*A. **Classical Sliding Mode Control:***

The classical sliding mode control is simulated for a two-link robotic manipulator whose parameter matrices are as follows [3].

$$\mathbf{M(q)} = \begin{bmatrix} (m_1+m_2)a_1^2 + m_2a_2^2 + 2m_2a_1a_2\cos(q_2) & m_2a_2^2 + m_2a_1a_2\cos(q_2) \\ m_2a_2^2 + m_2a_1a_2\cos(q_2) & m_2a_2^2 \end{bmatrix}$$

$$\mathbf{C(q,\dot{q})\dot{q}} = \begin{bmatrix} -m_2a_1a_2\left(2\dot{q}_1\dot{q}_2 + \dot{q}_2^2\right)\sin(q_2) \\ m_2a_1a_2\dot{q}_1^2\sin(q_2) \end{bmatrix}$$

$$\mathbf{G(q)} = \begin{bmatrix} (m_1+m_2)ga_1\cos(q_1) + m_2ga_2\cos(q_2+q_1) \\ m_2ga_2\cos(q_2+q_1) \end{bmatrix} \tag{16}$$

Where m_1 and m_2 are the mass, and a_1 and a_2 are the length of the links 1 and 2, respectively. They are chosen as m_1 = 0.8 Kg, m_2 =2.3Kg, a_1 =1m, a_2 =1m and g =9.8m/s^2. the control input $\boldsymbol{\tau}$ chosen as in (6), where $\boldsymbol{\lambda} = diag[1,1]$, $\mathbf{A} = diag[1,1]$, $\mathbf{K} = diag[20,10]$ and $\mathbf{q_d} = [\sin(t) \quad \cos(t)]^T$.In the simulation, the system model for the control input is estimated by applying a factor to the corresponding parameter matrices of the original system, i.e, $\hat{M}_{11} = 0.95M_{11}$, $\hat{M}_{12} = 0.95M_{12}$, $\hat{M}_{21} = 0.95M_{21}$, $\hat{M}_{22} = M_{22}$, $\hat{C} = C$ and $\hat{G} = 0.95G$.

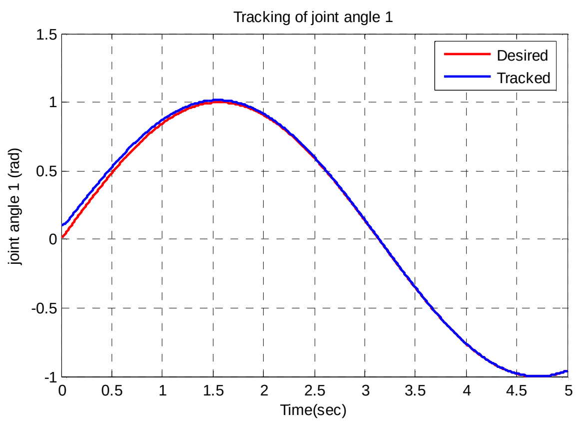

Fig.3. Tracking of joint 1 in the CSMC.

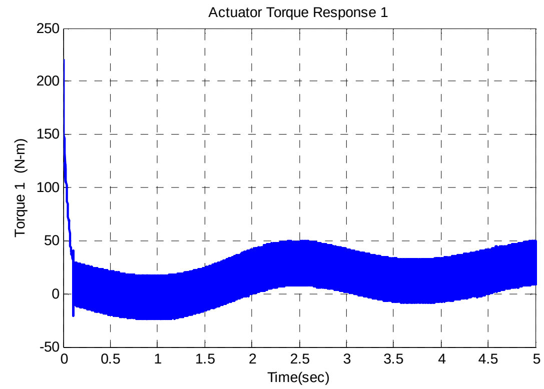

Fig.4. Actuator Torque Response of joint 1 in CSMC.

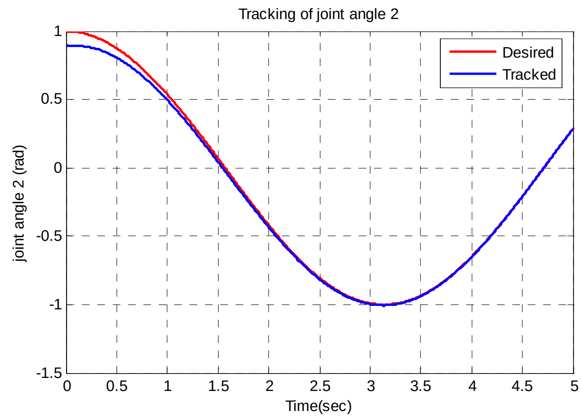

Fig.5. Tracking of joint 2 in the CSMC

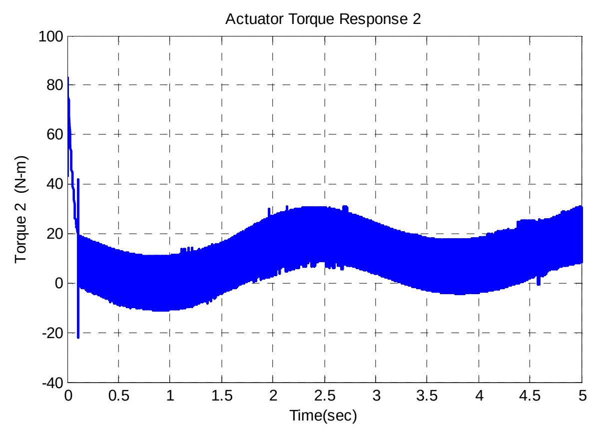

Fig.6. Actuator Torque Response of joint 2 in CSMC

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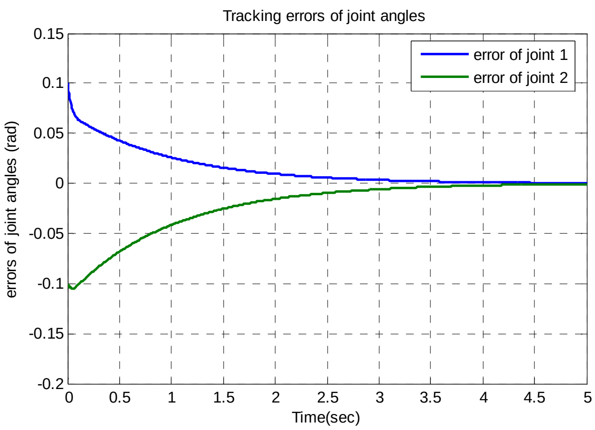

Fig.7. Tracking errors e_1 and e_2 in CSMC

B. Fuzzy and Higher Order Sliding Mode Control:

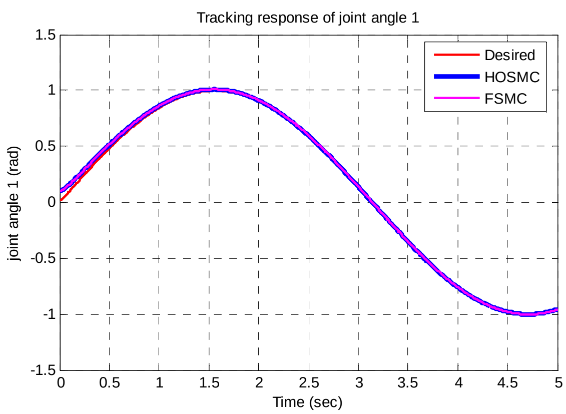

Fig.8. Tracking of joint 1 in FSMC and HOSMC.

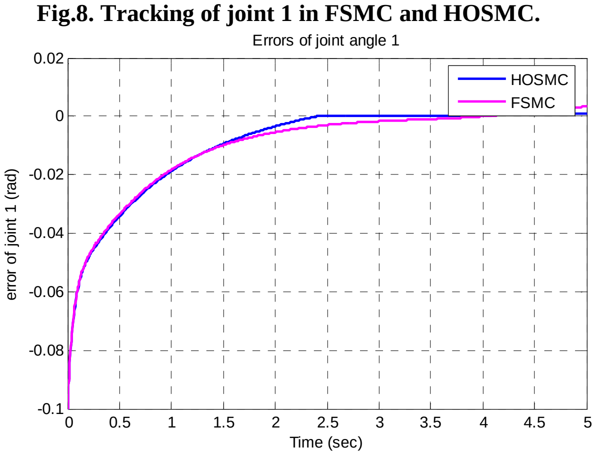

Fig.9. Tracking errors of joint 1 in FSMC and HOSMC

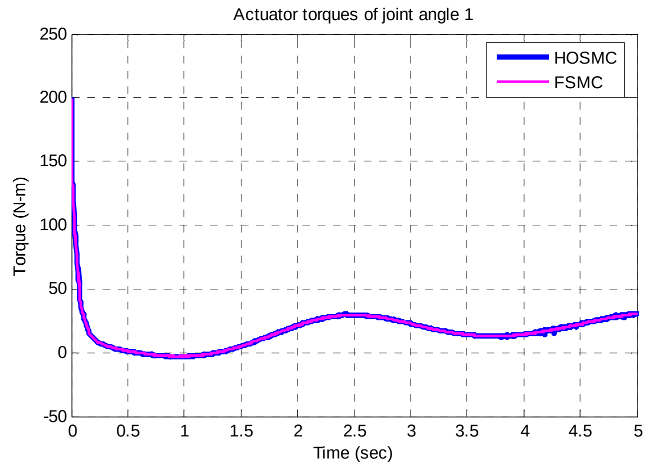

Fig.10. Actuator Torque Response of joint 1 in FSMC and HOSMC

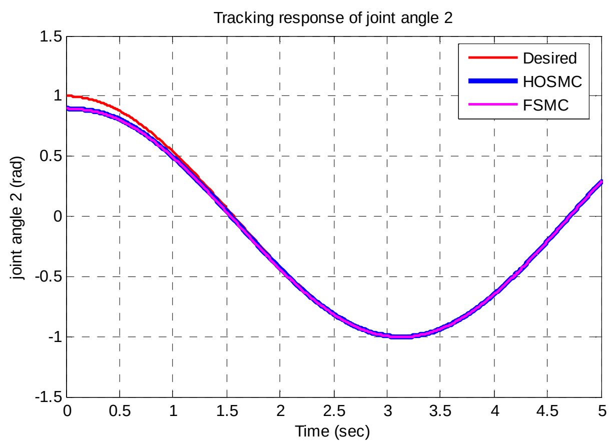

Fig.11. Tracking of joint 2 in FSMC and HOSMC

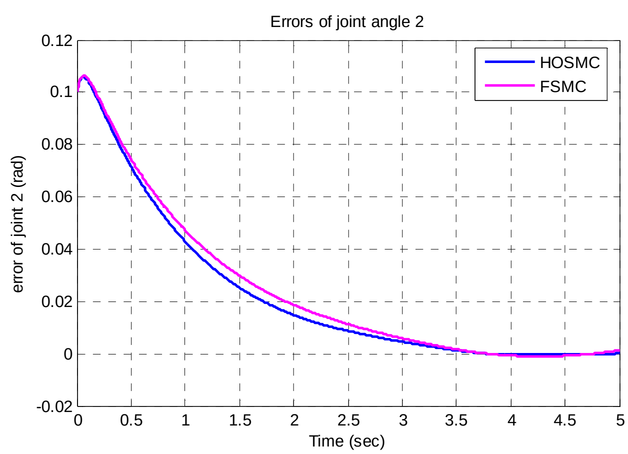

Fig.12. Tracking error of joint 2 in FSMC and HOSMC

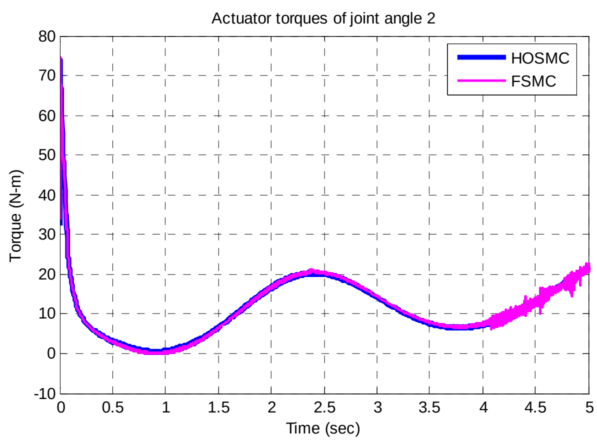

Fig.13. Actuator Torque Response of joint 2 in FSMC and HOSMC

Sl.No.	***Sliding mode Control(SMC) method***	***Tracking error***	***Chattering effect***
1	Conventional SMC	0	Severe
2	Fuzzy SMC	0.012	Removed Significantly
3	Higher Order SMC	0	Eliminated Completely

Conclusions

A higher order sliding mode controller for a two-link robot manipulator has been proposed. This approach provides robustness in the face of uncertainty. The performance of the higher order sliding mode controller is compared with a fuzzy sliding mode controller and conventional sliding mode controller on a nonlinear model of a two link robot. It is clear from the simulation results that the proposed control algorithm gives better performance. Further research work is being done in the area of discrete time higher order sliding mode control.

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